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FORECASTER, A MARKOVIAN MODEL TO ANALYZE THE DISTRIBUTION OF NAVAL OFFICERS

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A mathematical model is described for forecasting the distribution of officers in a Navy officer community at some future point in time. The distribution is in terms of activity types and tour numbers. The activity types stand for a categorization of the kind of billets Navy officers may occupy on successive tours of duty, such as sea and shore billets. The specific method of categorization is dictated by the community modeled and the purpose of the analysis. The emphasis here is on the mathematical derivation, although a numerical application is also included.

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FORECASTER, A MARKOVIAN MODEL TO ANALYZE THE DISTRIBUTION OF NAVAL OFFICERS

by
Paul R. Milch

1. INTRODUCTION AND BACKGROUND

In an earlier report [4] the author constructed an analytical model based on Semi-Markov Processes representing the career paths of military officers in general and Surface Warfare Officers (SWO's) of the U.S. Navy in particular. The model was based on a rectangular grid representation of the entire career structure of a group of officers. In the rectangular grid, rows represent the various **activity types** (e.g., sea duties and shore duties) in which an officer of the group whose career structure is modeled may be engaged from time to time. Columns of the grid stand for the succession of **tours** that makes up the career of any military officer. In this way, an officer's career may be represented by a path through a number of nodes (billets) whose first coordinates show the activity types and the second coordinates the tour numbers.

Associated with each node on the grid is a specific non-negative integer, called the **tourlength**, which determines the length of time the officer spends in that node or billet. A zero tourlength means that the node or billet is infeasible. This way, a matrix of size A by R determines the **tourlength matrix** where A and R are, respectively, the number of activity types and the number of tours in the entire career structure. A similar A by R matrix may represent the number of officers, called **incumbents**, currently occupying billets in all the feasible nodes. Yet a third A by R matrix of non-negative integers provides the numbers of **accessions** or recruits entering the system at future times. Typically, this latter matrix is all zeros, except for some elements of the first column, because new entries into the system should start with the first tour. However, in some officer communities, such as the Navy Nurse corps, for example, there are significant numbers of "recalls" and other "lateral entries" who join the community in later tours. Finally, the progression of advancement on the career grid is governed by a sequence of **transition probability matrices** which prescribe the probability of an officer currently in a specific node (i.e., activity-tour combination) transiting to another (possibly the same) activity type upon finishing his/her current tour. These matrices are all A by A and there are one fewer of them than the total number of tours (i.e., R–1). A more detailed description of the system is given in Milch [4].

In the same report the mathematical details and formulas are also described for the expected numbers of officers occupying each node (billet) on the grid at some future point in time. This is based on first computing probabilities that an officer occupying a node (billet) now will be anywhere on the grid t time units later. This computation is based on a Semi-Markov Process representation of the position occupied by an officer at time t. When the appropriate formulas were programmed for computation in APL on the IBM 3033 mainframe computer at the NPS it was found that the computation required either excessively large amounts of memory or huge amounts of CPU time. This occurred even after various time-saving measures were effected. Since it was desirable to make the model available for personal computers as well, the necessity of revising the computational core became even more acute.

Simultaneously with efforts to revise the mathematical core of the model two attempts have been made to evaluate the practical usefulness of the model apart from its computational speed (or rather lack of it). Thus Johnson [3] built an interactive user friendly interface for the model now labeled FORECASTER, thereby greatly enhancing its usability. He also demonstrated how FORECASTER may be used by analyzing the currently topical issue of joint duty assignments effected by the recently passed Goldwater-Nichols Department of Defense Reorganization Act. Title IV of this act mandates the creation of so-called joint duty officers in all four military services and prescribes their professional education, as well as their use by the services. Johnson examined the effect of the new law on the SWO community: its ability to fill its "fair share" of joint duty billets and how that may affect the traditional SWO career path. More recently, Drescher [1] examined the same problem in another Navy officer community, namely that of Tactical Aviation pilots and flight officers. At the same time, Drescher reorganized the APL functions used in FORECASTER and provided a more thorough documentation for the model.

Concurrently with Drescher's work, this author has revised the mathematical core of the model. This new mathematical formulation, a conceptually much simpler one than the original presented in Milch [4] turned out to be significantly faster computationally as well and that is the subject of this report. It is also hoped that the new mathematical formulation of the model will make it easier to effect another significant improvement in the usefulness of the model. This issue has to do with the way the user must input data into the current model, as well as the way the user must make use of the output results.

Briefly summarized, this aspect is due to the fact that the model is based on a sequence of tours that the officers undergo. This is a realistic simulation of what actually happens, but it also means that incumbents data (i.e., numbers of officers currently occupying billets) must be categorized by activities and tours. Unfortunately, the usual method of recordkeeping of officers in any Navy community is not organized by tours, but rather by grade and years of commissioned service. Likewise, results of the model categorize officers distributed at some future time by activity and tour number. These results must then be compared to billet requirements that are usually not organized by tour number but by grade.

Johnson [3] and Drescher [1] demonstrated that the data *is* available by tour number from the source where all officer data originates, the Officer Master File. However, the fact remains that standard personnel planning by all Navy offices where such work is done is based on officer and billet data that are *not* organized by tour numbers and this practice is not likely to change in the near future. For this reason it is desirable to convert the model to accept input and produce output by grade and/or years of service rather than tour number in order to accommodate user needs.

This effort is currently under way. In the meantime, the new mathematical core was designed to accept data as the original model did which is described in Milch [4] and outlined at the beginning of this section. The next section describes the mathematical details of the model according to the newly created APL code available on either the IBM 3033 mainframe computer or a personal computer.

2. DATA PREPARATION

As a point of departure, the notation of the author's previous report [4] will be adopted wherever appropriate. Thus, the tourlength matrix, L, has the elements

$$\ell_{in} = \begin{array}{l}\text{duration of time spent in activity type i}\\ \text{by an officer on his/her } n^{th} \text{ tour,}\end{array}$$

for $i = 1, \ldots, A$ and $n = 1, \ldots, R$.

The incumbents matrix, Q, is defined as having elements

$$q_{in} = \begin{array}{l}\text{number of officers at time zero occupying}\\ \text{activity type i billets during their } n^{th} \text{ tour,}\end{array}$$

for $i = 1, \ldots, A$ and $n = 1, \ldots, R$.

Further, the accessions matrix, C, is defined as consisting of elements

$$c_{in} = \begin{array}{l}\text{number of officers accessed during a time period}\\ \text{directly into an activity type i and tour number n,}\end{array}$$

for $i = 1, \ldots, A$ and $n = 1, \ldots, R$.

Most of the time, matrix C will have positive elements only in its first column (for $n = 1$), since a newly accessed officer must usually start in tour number one. It is interesting to note, however, that this is *not* always the case. For example, officers are sometimes recalled shortly after their separation from the service or are laterally transferred from one community to another and may thus join the community being analyzed in a tour other than their first one.

Finally, the transition probability matrices, $P(n)$ are defined to have elements,

$p_{ij}(n)$ = probability that an officer transfers to an activity type j on his/her n+1st tour upon completion of his/her n^{th} tour in an activity type i,

for $i,j = 1, \ldots, A$ and $n = 1, \ldots, R-1$.

The construction of the model will be described in steps like those of an algorithm as that is most suitable for the procedure which turned out to be both simple and computationally efficient.

Step 1: Unravel the tourlength matrix, L, by columns, starting with the first column, followed by the second, etc., into a vector $\hat{\underline{\ell}}$. More precisely, define

$$\hat{\underline{\ell}} = (\hat{\ell}_1, \ldots, \hat{\ell}_K)$$

where

$$\hat{\ell}_k = \ell_{in} \quad \text{when} \quad k = (n-1)A+i$$

for $i = 1, \ldots, A$ and $n = 1, \ldots, R-1$ so that $k = 1, 2, \ldots, K = AR$

Step 2: Define the indices $k_1, k_2, \ldots, k_{K^*}$ as the first, second, ..., last values of k for which $\hat{\ell}_k$ is positive. These are called the "ascending ladder indices or epochs" by Feller [2]. Then the abbreviated version of $\hat{\underline{\ell}}$ may be defined by omitting its zero components. In other words,

$$\underline{\ell}^* = (\hat{\ell}_{k_1}, \ldots, \hat{\ell}_{k_{K^*}})$$

where

$$K^* = \sum_{n=1}^{R} \sum_{i=1}^{A} \delta_{in} \quad \text{with} \quad \delta_{in} = \begin{cases} 1 & \text{if } \ell_{in} > 0 \\ 0 & \text{if } \ell_{in} = 0 \end{cases}$$

It will be useful to define the partial sums of the components of

$$\underline{\ell}^* = (\ell^*_1, \ldots, \ell^*_{K^*})$$

as well. Let

$$s_0 = 0 \quad \text{and} \quad s_u = \sum_{j=1}^{u} \ell^*_j = \sum_{j=1}^{u} \hat{\ell}_{k_j} \quad \text{for } u = 1, \ldots, K^*$$

since $\ell^*_j = \hat{\ell}_{k_j}$. In Feller's terminology the s_u are the "ascending ladder heights."

Step 3: Next the vector $\underline{\ell}^*$ is expanded by repeating each of its components exactly as many times as indicated by its value. That is, if $\underline{\ell}^* = (3, 1, 5)$ then the expanded vector $\underline{\ell}^{**}$ will have nine components with the first three equal to 3, the fourth equal to 1 and the last five equal to 5. In general,

$$\underline{\ell}^{**} = (\ell^{**}_1, \ldots, \ell^{**}_{K^{**}})$$

where

$$K^* = \sum_{n=1}^{R} \sum_{i=1}^{A} \ell_{in}$$

The components of $\underline{\ell}^{**}$ may formally be defined as

$$\ell^{**}_j = \ell^*_{k_u} \quad \text{when} \quad s_{u-1} + 1 \leq j \leq s_u \quad \text{and} \quad u = 1, \ldots, K^*.$$

For example, when $u = 1$, $\ell^{**}_j = \ell^*_{k_1}$ for all $j = 1, \ldots, s_1 = \ell^*_{k_1}$. This means that for the above example $\ell^{**}_j = 3$, for $j = 1, 2, 3$.

Step 4: Next a similar procedure is followed with respect to the incumbents matrix, Q. First, Q is unraveled by columns into the vector $\hat{\underline{q}}$. That is,

$$\underline{\hat{q}} = (\hat{q}_1, \ldots, \hat{q}_K)$$

where

$$\hat{q}_k = q_{in} \quad \text{when} \quad k = (n-1)A+i$$

for $i = 1, \ldots, A$ and $n = 1, \ldots, R$ so that $k = 1, \ldots, K = AR$.

Then $\underline{\hat{q}}$ is abbreviated by the omission of those of its components, $\hat{q}_k$, for which the corresponding $\hat{\ell}_k$ is zero. That is,

$$\underline{q}^* = (q^*_1, \ldots, q^*_{K^*})$$

where $q^*_j = \hat{q}_{k_j}$ for $j = 1, \ldots, K^*$.

Note that the decision to keep or omit a component, $\hat{q}_k$, of $\underline{\hat{q}}$ is based *not* on whether $\hat{q}_k>0$ or $\hat{q}_k=0$, but on whether $\hat{\ell}_k>0$ or $\ell_k=0$.

Finally, the vector $\underline{q}^*$ is expanded to repeat each of its components as many times as indicated by the value of the corresponding component of $\underline{\ell}^*$. More formally,

$$\underline{q}^{**} = (q^{**}_1, \ldots, q^{**}_{K^{**}})$$

where

$$q^{**}_j = q^*_{k_u} \quad \text{when} \quad s_{u-1} + 1 \le j \le s_u \quad \text{and} \quad u = 1, \ldots, K^*.$$

Step 5: Next two "flow rate" vectors are defined by dividing the number of incumbents by their respective tourlengths:

$$\underline{r}^* = (r^*_1, \ldots, r^*_{K^*}) \quad \text{and} \quad \underline{r}^{**} = (r^{**}_1, \ldots, r^{**}_{K^{**}})$$

where

$$r^*_j = \frac{q^*_j}{\ell^*_j} \quad \text{for} \quad j = 1, \ldots, K^*$$

and

$$r^{**}_j = \frac{q^{**}_j}{\ell^{**}_j} \quad \text{for} \quad j = 1, \ldots, K^{**}.$$

Step 6: A slightly different procedure is followed with the accessions matrix, C. Again, C is first unraveled by columns into the vector

$$\hat{\underline{c}} = (\hat{c}_1, \ldots, \hat{c}_K)$$

where

$$\hat{c}_k = c_{in} \quad \text{when} \quad k = (n-1)A+i$$

for $i = 1, \ldots, A$ and $n = 1, \ldots, R$ so that $k = 1, \ldots, K = AR$.

Then $\hat{\underline{c}}$ is abbreviated by the omission of those of its components, $\hat{c}_k$, for which the corresponding $\hat{\ell}_k$ is zero. That is,

$$\underline{c}^* = (c^*_1, \ldots, c^*_{K^*})$$

where
$$c^*_j = \hat{c}_{k_j} \quad \text{for} \quad j = 1, \ldots, K^*.$$

Then $\underline{c}^*$ is expanded to have K^{**} components, but instead of repeating its components (as was the case in expanding $\underline{q}^*$ into $\underline{q}^{**}$) the extra components in $\underline{c}^{**}$ are simply zeroes. More precisely,

$$\underline{c}^{**} = (c^{**}_1, \ldots, c^{**}_{K^{**}})$$

where

$$c_j^{**} = \begin{cases} c_{k_u}^{*} & \text{when } \; j = s_{u-1} + 1 \; \text{ and } \; u = 1, \ldots, K^* \\ 0 & \text{otherwise.} \end{cases}$$

These six steps accomplish the task of rearranging the data originally organized for user convenience into formats that fit the need of efficient storage and computation. Namely, the tourlength matrix, L, was manipulated first to eliminate infeasible billet types and then to repeat its positive components in preparation of the computation of the flow rate vectors $\underline{r}^*$ and $\underline{r}^{**}$. Similarly, the incumbents matrix was first transformed into a vector in which any incumbents occupying infeasible billets (due to a data recording error or for any other reasons) have been omitted. The practical effect of this is that such personnel will be attrited by the model during the forecasting process which is probably the appropriate representation of what happens to such personnel, anyway.

The purpose of dividing the number of incumbents by the corresponding tourlength and then repeating that quotient as many times as indicated by the value of the tourlength is to distribute personnel occupying a billet type *evenly* in terms of longevity (or experience) in that billet. This may not be the same as the actual distribution in reality, but the latter may either not be available or would present too much trouble for the user to enter into the model for the additional accuracy that would be gained thereby.

The accessions matrix is similarly treated except that instead of repeating its components all accessions are placed into the first (i.e., zero) longevity (or experience) category and the additional components of $\underline{c}^{**}$ are simply zeroes.

The next task is to consolidate the transition probability matrices, P(n), n = 1, ..., R-1, into a single large transition probability matrix that will affect the transitions of personnel occurring each unit of time. Conceptually the simplest way of accomplishing this would be to construct a Markov Chain with K** states corresponding to the components of the vector $\underline{r}^{**}$. Then each unit of time components of $\underline{r}^{**}$ would be transferred in accordance with a K** by K** transition probability matrix which would contain all the positive elements of all the P(n) matrices, but would otherwise consist of mostly zeroes and many ones, representing the fact that personnel cannot move backward (or "sideways") in time and once placed in a billet personnel must, by necessity, move up by one longevity (experience) level per unit of time until their experience level equals the tourlength in that billet at which point the appropriate elements of the relevant P(n) matrix would distribute them into various activities of their next tour.

The problem with this approach is that K**, being the sum of all tourlengths, is in the range of about 300 or more for most problems for which this model is being designed when tourlengths are given in terms of quarters (3 months) which is probably the preferred unit of time, with the alternative being months (which in turn would further increase the number of states three-fold). When forecasting several units of time into the future having the number of states in the neighborhood of 300 would tax the computational capabilities of most micro-computers in terms of memory and/or CPU time and perhaps even of most commonly available mainframe computers for which this model could be designed. Therefore, practicality dictates a better approach which is less consumptive of CPU time as well as memory.

The problem of too high dimensionality will be solved by using the Markov Chain approach not to forecast the vector $\underline{r}^{**}$ but the much lower dimensional vector $\underline{r}^{*}$. This reduces the dimensions from K^{**} (approximately 300) to K^{*} (approximately 50 or 60). This requires the transition probability matrix to be of only K^{*} by K^{*}.

It is this matrix that is constructed next from the sequence of transition probability matrices $P(n)$, $n = 1, \ldots, R-1$. This is accomplished in several steps.

First the matrices, $P(n)$, are made consistent with the information contained in the tourlength matrix, L. Namely, if the element l_{in} is zero, indicating that activity type i for tour number n is *not* feasible then the i^{th} row of the matrix $P(n)$ is superfluous and in fact should contain all zeroes. If some elements, $p_{ij}(n)$, are positive for some $j = 1, \ldots, A$ it must be due to some error, most likely an omission to change these elements to zero by a user of the model when a formerly feasible billet was made infeasible. At any rate, $\ell_{in} = 0$ implies that $P_{ij}(n)$ should be zero for all $j = 1, \ldots, A$ and therefore these elements occupy redundant space in $P(n)$. Similarly, the i^{th} column of the matrix $P(n-1)$ should contain all zero elements and is likewise redundant. This results in the following

Step 1: Define $\hat{P}(n)$, for $n = 1, \ldots, R-1$, to have elements

$$\hat{p}_{ij}(n) = \begin{cases} p_{ij}(n) & \text{if} \quad \ell_{in}\ell_{jn+1} > 0 \\ 0 & \text{if} \quad \ell_{in}\ell_{jn+1} = 0 \end{cases} \qquad \text{for } i,j = 1, \ldots, A.$$

Then define the matrix $P^{*}(n)$, for $n = 1, \ldots, R-1$, to be the same as $\hat{P}(n)$ except that any all-zero row or all-zero column is omitted. Note that the matrices, $P^{*}(n)$, may no longer be square matrices and in fact it will be useful to introduce notation for their row and column dimensions. Namely, let

$$dr(n) = \text{no. of rows of } P^*(n)$$

$$dc(n) = \text{no. of columns of } P^*(n)$$

for $n = 1, \ldots, R-1$.

Also, let $dr(0) = dc(0) = 0$.

Further, define the partial sums of these row and column dimensions:

$$Dr(n) = \sum_{m=0}^{n} dr(m)$$

$$Dc(n) = \sum_{m=0}^{n} dc(m)$$

for $n = 0, 1, \ldots, R-1$.

Note that

$$dr(n) = \sum_{i=1}^{A} \delta_{in} \quad \text{and} \quad dc(n) = \sum_{i=1}^{A} \delta_{in+1}$$

for $n = 1, \ldots, R-1$.

Therefore, $dc(n) = dr(n+1)$ or the column dimension of $P^*(n)$ is the same as the row dimension of $P^*(n+1)$.

Also, note that

$$K^* = \sum_{n=1}^{R} \sum_{i=1}^{A} \delta_{in} = \sum_{n=1}^{R-1} dr(n) + \sum_{i=1}^{A} \delta_{iR} = Dr(R-1)+dc(R-1)$$

and similarly,

$$K^* = \sum_{n=0}^{R-1} \sum_{i=1}^{A} \delta_{in+1} = \sum_{i=1}^{A} \delta_{i1} + \sum_{n=1}^{R-1} dc(n) = dr(1)+Dc(R-1)$$

Therefore,

$$Dr(R-1) + dc(R-1) = dr(1) + Dc(R-1)$$

or

$$Dr(R-1) - dr(1) = Dc(R-1) - dc(R-1)$$

That is, the sum of all but the first row dimensions equals the sum of all but the last column dimensions.

Step 2: The K* by K* dimensional transition probability matrix, P*, may now be introduced with all elements zero except the following:

$$p^*_{Dr(n-1)+i,\ dr(1)+Dc(n-1)+j} = p^*_{ij}(n)$$

for $i = 1, \ldots, dr(n)$ and $j = 1, \ldots, dc(n)$ and $n = 1, \ldots, R-1$.

Note that by necessity the first dr(1) columns of the matrix, P*, must be all zeroes. Similarly, the last dc(R–1) rows of the matrix P*, are also all zeroes. It is also clear that P* is an upper triangular matrix, in fact all its main diagonal elements are also zero.

The following example may serve as an illustration of the entire procedure described above. The three matrices, L, Q and C are given as

$$L = \begin{pmatrix} 2 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 2 & 0 & 4 \end{pmatrix} \qquad Q = \begin{pmatrix} 50 & 0 & 60 & 0 \\ 0 & 10 & 8 & 50 \\ 90 & 30 & 20 & 80 \end{pmatrix}$$

and

$$C = \begin{pmatrix} 70 & 0 & 10 & 0 \\ 10 & 0 & 5 & 0 \\ 50 & 0 & 0 & 0 \end{pmatrix}$$

with $i = 3$ and $R = 4$.

Then the unraveling process produces the $K=12$ dimensional vectors

$$\hat{\underline{\ell}} = (2, 0, 3, 0, 1, 2, 3, 0, 0, 0, 2, 4)$$

$$\hat{\underline{q}} = (50, 0, 90, 0, 10, 30, 60, 8, 20, 0, 50, 80)$$

$$\hat{\underline{c}} = (70, 10, 50, 0, 0, 0, 10, 5, 0, 0, 0, 0)$$

Examining $\hat{\underline{\ell}}$ shows that the "ascending ladder epochs" are:

$$k_1 = 1,\ k_2 = 3,\ k_3 = 5,\ k_4 = 6,\ k_5 = 7,\ k_6 = 11,\ k_7 = 12.$$

Therefore, the abbreviated vectors are $K^* = 7$ dimensional, namely:

$$\underline{\ell}^* = (2, 3, 1, 2, 3, 2, 4)$$

$$\underline{q}^* = (50, 90, 10, 30, 60, 50, 80)$$

$$\underline{c}^* = (70, 50, 0, 0, 10, 0, 0)$$

The "ascending ladder heights" are:

$$s_0 = 0,\ s_1 = 2,\ s_2 = 5,\ s_3 = 6,\ s_4 = 8,\ s_5 = 11,\ s_6 = 13,\ s_7 = 17.$$

The vectors with repeated components are $K^{**} = 17$ dimensional, namely:

$$\underline{\ell}^{**} = (2, 2, 3, 3, 3, 1, 2, 2, 3, 3, 3, 2, 2, 4, 4, 4, 4)$$

$$\underline{q}^{**} = (50, 50, 90, 90, 90, 10, 30, 30, 60, 60, 60, 50, 50, 80, 80, 80, 80)$$

$$\underline{c}^{**} = (70, 0, 50, 0, 0, 0, 0, 0, 10, 0, 0, 0, 0, 0, 0, 0, 0)$$

Next, the two flow rate vectors of dimensions $K^* = 7$ and $K^{**} = 17$ are computed as:

$$\underline{r}^* = (25, 30, 10, 15, 20, 25, 20)$$

$$\underline{r}^{**} = (25, 25, 30, 30, 30, 10, 15, 15, 20, 20, 20, 25, 25, 20, 20, 20, 20)$$

Assume that the following three P(n), n = 1, 2, 3, matrices are also given:

$$P(1) = \begin{pmatrix} 0 & .5 & .4 \\ .1 & .7 & 0 \\ .0 & .8 & .1 \end{pmatrix} \quad P(2) = \begin{pmatrix} .8 & 0 & .1 \\ .8 & 0 & 0 \\ .7 & .1 & 0 \end{pmatrix} \quad P(3) = \begin{pmatrix} .2 & .6 & .1 \\ .2 & .5 & .2 \\ .8 & .1 & 0 \end{pmatrix}.$$

The first step is to replace any positive $p_{ij}(n)$'s with zeroes if either $\ell_{in} = 0$ or $\ell_{jn}=0$.

$$\hat{P}(1) = \begin{pmatrix} 0 & .5 & .4 \\ 0 & 0 & 0 \\ 0 & .8 & .1 \end{pmatrix} \quad \hat{P}(2) = \begin{pmatrix} 0 & 0 & 0 \\ .8 & 0 & 0 \\ .7 & 0 & 0 \end{pmatrix} \quad \hat{P}(1) = \begin{pmatrix} 0 & .6 & .1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Next, all-zero rows and all-zero columns are eliminated:

$$P^*(1) = \begin{pmatrix} .5 & .4 \\ .8 & .1 \end{pmatrix} \quad P^*(2) = \begin{pmatrix} .8 \\ .7 \end{pmatrix} \quad P^*(3) = (.6 \quad .1)$$

Note that

$$dr(1) = 2, dr(2) = 2, dr(3) = 1 \text{ and } dc(1) = 2, dc(2) = 1, dc(3) = 2.$$

Therefore,

$$Dr(0) = 0, Dr(1) = 2, Dr(2) = 4, Dr(3) = 5$$

and $$Dc(0) = 0, Dc(1) = 2, Dc(2) = 3, Dc(3) = 5.$$

The $K^*=7$ dimensional square matrix P^* is

$$P^* = \begin{pmatrix} 0 & 0 & .5 & .4 & 0 & 0 & 0 \\ 0 & 0 & .8 & .1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .8 & 0 & 0 \\ 0 & 0 & 0 & 0 & .7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .6 & .1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Note the way the matrices, $P^*(1)$, $P^*(2)$ and $P^*(3)$, are embedded in the large P^* matrix, in the form of a "descending staircase" which is typical of the structure of the P^* matrix.

3. FORECASTING

Forecasting will be accomplished in several steps using both K^*- and K^{**}-dimensional vectors.

The first five of these steps are taken for a single value of t, starting with $t = 0$ and then repeated for $t = 1, \ldots, T-1$ where T is the forecasting horizon for which the distribution of officers is required. After the first five steps have been successfully repeated for $t = 0, 1, \ldots, T-1$ the actual forecasting procedure is complete. Steps 6, 7, and 8 then serve the purpose of rearranging the result in the standard A by R matrix format.

Step 1: First the K*-dimensional vector $\underline{r}^*(t)$ is established, made up of those elements of $\underline{r}^{**}(t)$ that stand for the highest "experience" level in each billet. More precisely, using the "ascending ladder height,"

$$\underline{r}^*(t) = (r_1^*(t), \ldots, r_{K^*}^*(t))$$

where

$$r_u^*(t) = r_{s_u}^{**}(t) \text{ for } u = 1, 2, \ldots, K^*$$

with $\underline{r}^{**}(0) = \underline{r}^{**}$ as given in the previous section.

Step 2: Update $\underline{r}^*(t)$ by the usual Markovian equation of

$$\underline{r}^{**}(t+1) = \underline{r}^*(t)P^*$$

Step 3: Update $\underline{r}^{**}(t)$ by "shifting" its components one component to the right, that is,

$$r_j^{**}(t+1) = \begin{cases} 0 & \text{if } j = 1 \\ r_{j-1}^{**}(t) & \text{if } j = 2, \ldots, K^{**} \end{cases}$$

with $\underline{r}^{**}(0) = \underline{r}^{**}$ as in Step 1.

Step 4: Replace those components of the updated vector, $\underline{r}^{**}(t+1)$, which stand for zero "experience" level in each billet by successive components of the updated vector $\underline{r}^{*}(t+1)$. More precisely,

$$r^{**}_{s_u+1}(t+1) = r^{*}_{u+1}(t+1) \quad \text{for} \quad u = 0, \ldots, K^*-1.$$

Step 5: As a last step in the updating process the accession vector, $\underline{c}^{**}$ is added to the vector $\underline{r}^{**}(t+1)$ obtained in Step 4. That is,

$$\text{new } \underline{r}^{**}(t+1) = \text{old } r^{**}(t+1) + \underline{c}^{**}$$

These five steps are now repeated for the next higher t-value until $\underline{r}^{**}(T)$ is obtained. Then Steps 6, 7, and 8 follow:

Step 6: To add the appropriate components of $\underline{r}^{**}(T)$ is easiest accomplished in terms of the "ascending ladder heights," s_u. Namely, define

$$\underline{q}^*(T) = (q^*_1(T), \ldots, q^*_{K^*}(T)) \quad \text{for} \quad T = 1, 2, \ldots$$

where

$$q^*_{u+1}(T) = \sum_{j=s_u+1}^{s_{u+1}} r^{**}_j(T) \quad \text{for} \quad u = 0, 1, \ldots, K^*-1$$

Step 7: The vector $\underline{q}^*(T)$ is expanded to insert zeroes at the appropriate places, representing infeasible billets. This is accomplished in terms of the "ascending ladder epochs," k_j. Define

$$\hat{\underline{q}}(T) = (\hat{q}_1(T), \ldots, \hat{q}_K(T))$$

where

$$\hat{q}_k(T) = \begin{cases} q_j^*(T) & \text{when } k = k_j \ \text{ for } \ j = 1, \ldots, K^* \\ 0 & \text{otherwise.} \end{cases}$$

Step 8: Finally, the $\hat{\underline{q}}(T)$ is re-raveled into the original data format of an A by R matrix. That is, the matrix Q(T) is defined with elements

$$q_{in}(T) = \hat{q}_k(T)$$

where $k = (n-1)A+i$, with $i = 1, \ldots, A$ and $n = 1, \ldots, R$.

The numerical example introduced at the end of the previous section is continued to illustrate the procedure outlined above. The forecasting horizon of T=2 will be used.

In Step 1, the "ascending ladder heights"

$$s_0 = 0,\ s_1 = 2,\ s_2 = 5,\ s_3 = 6,\ s_4 = 8,\ s_5 = 11,\ s_6 = 13,\ s_7 = 17$$

are used to find the $K^* = 7$ dimensional vector

$$\underline{r}^*(0) = (25, 30, 10, 15, 20, 25, 20).$$

In Step 2, Markovian forecasting is applied to this vector using the 7 by 7 P^*-matrix given at the very end of the previous section. Thus,

$$\underline{r}^*(1) = \underline{r}^*(0)P^* = (0, 0, 36.5, 13, 18.5, 12, 2).$$

In Step 3, the $K^{**} = 17$ dimensional vector, $\underline{r}^{**}(0) = \underline{r}^{**}$ is shifted one component to the right resulting in

$$\underline{r}^{**}(1) = (0, 25, 25, 30, 30, 30, 10, 15, 15, 20, 20, 20, 25, 25, 20, 20, 20).$$

Next, in Step 4, using the "ascending ladder heights," again, appropriate components of $\underline{r}^{**}(1)$ above are replaced by successive components of $\underline{r}^{**}(1)$ computed in Step 2. The result is

$$\downarrow \quad \downarrow \qquad \downarrow \quad \downarrow \qquad \downarrow \qquad \downarrow \quad \downarrow$$

$$\underline{r}^{**}(1) = (0, 25, 0, 30, 30, 36.5, 13, 15, 18.5, 20, 20, 12, 25, 2, 20, 20, 20)$$

where the replaced components are marked by an arrow above them.

In Step 5, the accession vector

$$\underline{c}^{**} = (70, 0, 50, 0, 0, 0, 0, 0, 10, 0, 0, 0, 0, 0, 0, 0, 0)$$

is added to the result of Step 4 to obtain

$$\underline{r}^{**}(1) = (70, 25, 50, 30, 30, 36.5, 13, 15, 28.5, 20, 20, 12, 25, 2, 20, 20, 20).$$

Next, Steps 1 through 5 are repated for t = 1 with the following results:

Step 1: $\underline{r}^{*}(1) = (25, 30, 36.5, 15, 20, 25, 20)$

Step 2: $\underline{r}(2) = \underline{r}^{*}(1)P^{*} = (0, 0, 36.5, 13, 39.7, 12, 2)$

Step 3: $\underline{r}^{**}(2) = (0, 70, 25, 50, 30, 30, 36.5, 13, 15, 28.5, 20, 20, 12, 25, 2, 20, 20)$

$$\downarrow \quad \downarrow \qquad \downarrow \quad \downarrow \qquad \downarrow \qquad \downarrow \quad \downarrow$$

Step 4: $\underline{r}^{**}(2) = (0, 70, 0, 50, 30, 36.5, 13, 13, 39.7, 28.5, 20, 12, 12, 2, 2, 20, 20)$

Step 5: new $\underline{r}^{**}(2) = (70, 70, 50, 50, 30, 36.5, 13, 13, 49.7, 28.5, 20, 12, 12, 2, 2, 20, 20)$

Since $\underline{r}^{**}(T)$ has been obtained, it remains only to reformat the result.

In Step 6, the "ascending ladder heights," are used again to compute components of the vector

$$\underline{q}^{*}(2) = (140,\ 130,\ 36.5,\ 26,\ 98.2,\ 24,\ 44).$$

The details are as follows:

$$q_1^{*}(2) = \sum_{j=1}^{2} r_j^{**}(2) = 70 + 70 = 140 \qquad q_2^{*}(2) = \sum_{j=3}^{5} r_j^{**}(2) = 50 + 50 + 30 = 130$$

$$q_3^{*}(2) = \sum_{j=6}^{6} r_j^{**}(2) = 36.5 \qquad q_4^{*}(2) = \sum_{j=7}^{8} r_j^{**}(2) = 13 + 13 = 26$$

$$q_5^{*}(2) = \sum_{j=9}^{11} r_j^{**}(2) = 49.7 + 28.5 + 20 = 98.2 \qquad q_6^{*}(2) = \sum_{j=12}^{13} r_j^{**}(2) = 12 + 12 = 24$$

and $$q_7^*(2) = \sum_{j=14}^{17} r_j^{**}(2) = 2 + 2 + 20 + 20 = 44.$$

In Step 7, the "ascending ladder epochs,"

$$k_1 = 1,\ k_2 = 3,\ k_3 = 5,\ k_4 = 6,\ k_5 = 7,\ k_6 = 1$$

are used to expand the $K^* = 7$ dimensional vector $\underline{q}^*(2)$ into the $K=12$ dimensional vector

$$\hat{\underline{q}}(2) = (140,\ 0,\ 130,\ 0,\ 36.5,\ 26,\ 98.2,\ 0,\ 0,\ 0,\ 24,\ 44).$$

Finally, in Step 8, this vector is re-raveled into the 3 by 4 matrix

$$Q(2) = \begin{pmatrix} 140 & 0 & 98.2 & 0 \\ 0 & 36.5 & 0 & 24 \\ 130 & 26 & 0 & 44 \end{pmatrix}.$$

In practice, each element of this matrix would likely be rounded to the nearest integer if this is indeed the end of the forecasting process, as these are the expected numbers of officers two quarters into the future.

4. AN APPLICATION.

A typical application of the model to the Surface Warfare Officer (SWO) community of the U.S. Navy follows. The choice of activity types, listed in Table 1, was dictated by both the nature of the community and the topical issue of "jointness" referred to in the Introduction and Background. Thus, to analyze the influence of the new legislation on SWO career paths the activity types of "JPME" (Joint Professional Military Education) and "Joint Tours," were broken out separately in addition to "Postgraduate Education" and "SWO Education" (such as Department Head School) from the other "Shore Duty," with "Fleet Unit" being the main activity why the community exists.

The tourlength matrix, L, the Incumbents matrix, Q, and the Accessions matrix, C, are given in Tables 2, 3, and 4, respectively. Finally, the eleven transition probability matrices are presented in Table 5.

The model was then used to forecast the distribution of SWOs four, ten and forty quarters into the future with Tables 6, 7 and 8 showing the results. The CPU-times required to perform these calculations were, respectively, 7.69, 16.31, and 59.1 seconds on a Zenith Z-248(R) microcomputer with 1.1 MB RAM.

The model's user-friendly features which enable the user to effect changes in any of the data input values given in Tables 1 through 5 and to carry out a billets versus officers comparison in a reasonably efficient manner are detailed in Johnson [3] and are not discussed here.

TABLE 1. CURRENT LIST OF ACTIVITY NAMES IN THE SURFACE COMMUNITY

1. POSTGR EDUC
2. JPME
3. JOINT TOUR
4. SWO EDUC
5. FLEET UNIT
6. SHORE DUTY

TABLE 2. CURRENT TOUR LENGTHS IN QUARTERS

ACTIVITIES/TOURS	1	2	3	4	5	6	7	8	9	10	11	12
1. POSTGR EDUC	0	8	8	8	6	6	6	6	6	0	6	0
2. JPME	0	0	2	0	2	2	2	2	2	2	2	2
3. JOINT TOUR	0	8	8	8	8	8	8	8	8	8	8	8
4. SWO EDUC	0	2	2	2	2	2	0	0	0	0	0	0
5. FLEET UNIT	12	6	6	6	6	6	6	6	6	9	9	9
6. SHORE DUTY	0	8	8	8	8	8	8	8	8	8	8	8

TABLE 3. CURRENT INCUMBENTS

ACTIVITIES/TOURS	1	2	3	4	5	6	7	8	9	10	11	12
1. POSTGR EDUC	0	124	133	12	12	28	8	7	2	0	2	0
2. JPME	0	0	3	0	2	11	9	12	15	7	4	1
3. JOINT TOUR	0	1	3	3	3	12	15	36	91	41	39	19
4. SWO EDUC	0	24	55	69	15	9	0	0	0	0	0	0
5. FLEET UNIT	3744	765	168	299	465	253	225	194	313	172	69	46
6. SHORE DUTY	0	466	530	126	88	119	117	244	428	252	137	83

TABLE 4. CURRENT ACCESSIONS

ACTIVITIES/TOURS	1	2	3	4	5	6	7	8	9	10	11	12
1. POSTGR EDUC	0	0	0	0	0	0	0	0	0	0	0	0
2. JPME	0	0	0	0	0	0	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0	0	0	0	0	0	0	0
4. SWO EDUC	0	0	0	0	0	0	0	0	0	0	0	0
5. FLEET UNIT	315	0	0	0	0	0	0	0	0	0	0	0
6. SHORE DUTY	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 5. CURRENT TRANSFER RATES

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 1

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	0	0
2. JPME	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0.05	0	0	0.05	0.40	0.18
6. SHORE DUTY	0	0	0	0	0	0

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 2

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	1.00	0	0
2. JPME	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0	0
4. SWO EDUC	0	0	0	0	1.00	0
5. FLEET UNIT	0.13	0	0	0	0.10	0.52
6. SHORE DUTY	0	0	0	0.24	0	0

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 3

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	1.00	0	0
2. JPME	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0	0
4. SWO EDUC	0	0	0	0	1.00	0
5. FLEET UNIT	0	0	0	0	1.00	0
6. SHORE DUTY	0	0	0	0.33	0	0.25

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 4

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	0	0
2. JPME	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0	0
4. SWO EDUC	0	0	0	0	1.00	0
5. FLEET UNIT	0.05	0	0	0	0.80	0.15
6. SHORE DUTY	0	0	0	0.50	0	0

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 5

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	1.00	0
2. JPME	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0	0
4. SWO EDUC	0	0	0	0	1.00	0
5. FLEET UNIT	0.05	0.07	0.02	0	0.36	0.40
6. SHORE DUTY	0	0	0	0	0.80	0

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 6

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	1.00	0
2. JPME	0	0	0.40	0	0.60	0
3. JOINT TOUR	0	0	0	0	0.85	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0.05	0.10	0.04	0	0.40	0.40
6. SHORE DUTY	0	0	0	0	0.85	0

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 7

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	1.00	0
2. JPME	0	0	0.50	0	0.50	0
3. JOINT TOUR	0	0	0	0	1.00	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0.05	0.15	0.05	0	0.25	0.50
6. SHORE DUTY	0	0	0	0	1.00	0

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 8

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	1.00	0
2. JPME	0	0	0.90	0	0.10	0
3. JOINT TOUR	0	0	0	0	1.00	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0	0.15	0.15	0	0.25	0.45
6. SHORE DUTY	0	0	0	0	0.65	0.35

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 9

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	0	0
2. JPME	0	0	0.50	0	0.50	0
3. JOINT TOUR	0	0	0	0	1.00	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0	0.06	0.02	0	0.15	0.65
6. SHORE DUTY	0	0	0	0	.45	0.25

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 10

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	0	0
2. JPME	0	0	0.50	0	0.50	0
3. JOINT TOUR	0	0	0	0	0.95	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0	0	0.10	0	0.15	0.40
6. SHORE DUTY	0	0	0	0	0.20	0.30

CURRENT TRANSFER RATES WHEN LEAVING TOUR NUMBER 11

ACTIVITIES/ACTIVITIES	1	2	3	4	5	6
1. POSTGR EDUC	0	0	0	0	0	0
2. JPME	0	0	0	0	0	0
3. JOINT TOUR	0	0	0	0	0.80	0
4. SWO EDUC	0	0	0	0	0	0
5. FLEET UNIT	0	0	0.20	0	0	0.35
6. SHORE DUTY	0	0	0	0	0.30	0.35

TABLE 6. EXPECTED NUMBERS OF OFFICERS 4 QUARTERS FROM NOW

ACTIVITIES/TOURS	1	2	3	4	5	6	7	8	9	10	11	12
1. POSTGR EDUC	0	126	135	8	14	26	10	10	0	0	0	0
2. JPME	0	0	0	0	0	11	8	11	10	6	0	0
3. JOINT TOUR	0	0	0	0	0	14	24	37	84	37	35	14
4. SWO EDUC	0	31	60	78	16	0	0	0	0	0	0	0
5. FLEET UNIT	3756	755	162	328	464	272	235	183	243	280	103	61
6. SHORE DUTY	0	457	530	130	74	185	127	200	317	317	137	75

TABLE 7. EXPECTED NUMBERS OF OFFICERS 10 QUARTERS FROM NOW

ACTIVITIES/TOURS	1	2	3	4	5	6	7	8	9	10	11	12
1. POSTGR EDUC	0	125	131	0	16	23	14	12	0	0	0	0
2. JPME	0	0	0	0	0	11	9	12	9	4	0	0
3. JOINT TOUR	0	0	0	0	0	12	32	33	79	26	29	14
4. SWO EDUC	0	31	58	77	17	0	0	0	0	0	0	0
5. FLEET UNIT	3774	749	169	342	495	276	267	192	193	373	150	80
6. SHORE DUTY	0	449	526	132	64	248	142	155	190	319	158	73

TABLE 8. EXPECTED NUMBERS OF OFFICERS 40 QUARTERS FROM NOW

ACTIVITIES/TOURS	1	2	3	4	5	6	7	8	9	10	11	12
1. POSTGR EDUC	0	126	131	0	17	25	14	17	0	0	0	0
2. JPME	0	0	0	0	0	12	10	17	13	4	0	0
3. JOINT TOUR	0	0	0	0	0	13	34	42	113	31	34	20
4. SWO EDUC	0	32	59	76	16	0	0	0	0	0	0	0
5. FLEET UNIT	3780	756	170	346	505	289	340	255	229	311	132	73
7. SHORE DUTY	0	454	524	131	69	269	153	227	232	243	166	84

LIST OF REFERENCES

[1] Drescher, Richard B., "The Effect of the Goldwater-Nichols Department of Defense Reorganization Act on Tactical Aviation Pilot and NFO Career Paths," Master's Thesis, Naval Postgraduate School, Monterey, CA, September 1989.

[2] Feller, W., *An Introduction to Probability Theory and Its Applications,* Volume II, John Wiley, 1966, pp. 189-193 and 374-377.

[3] Johnson, Joseph L., Jr., "Analysis of the Joint Reorganization Act's Impact on Personnel Flow in the Surface Warfare Officer Community," Master's Thesis, Naval Postgraduate School, Monterey, CA, March 1989.

[4] Milch, Paul R. (1988). "An Analytical Model for Forecasting Navy Officer Career Paths," NPS Technical Report #NPS55-88-009, September 1988.

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